UNIVERSITI SAINS MALAYSIA

Final Examination
Academic Session 2007/2008

April 2008

JIM 211 – Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains SEVEN printed pages before you begin the examination.

Answer ALL questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.]

1. (a) Form the difference quotients

$$\frac{f(x+h,y)-f(x,y)}{h} \text{ and } \frac{f(x,y+h)-f(x,y)}{h},\ (h\neq 0).$$

Then, assuming that x and y are fixed, calculate the limit as $h\to 0$.

$f(x,y)=2x^2-y$.

(25 marks)

(b) Identify the surface and find the traces:

$$9x^2+4y^2-36z=0.$$

(25 marks)

(c) Find an equation for the level curve of f that contains the point P.

$$f(x,y)=1-4x^2-y^2;\quad P(0,1).$$

(25 marks)

(d) Calculate the partial derivatives:

$$f(x,y,z)=z\sin(x-y).$$

(25 marks)

2. (a) Show that, if $u=\dfrac{xy}{x+y}$ then $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=0$.

(25 marks)

(b) Find the gradient vector at the point P.

$$f(x,y)=2x^2-3xy+4y^2 \text{ at } P(2,3).$$

(25 marks)

(c) Find $\dfrac{\partial u}{\partial s}$ and $\dfrac{\partial u}{\partial t}$.

$u=x^2-xy$

where $x=s\cos t,\quad y=t\sin s$.

(25 marks)

(d) Find the point(s) on the surface at which the tangent plane is horizontal.

$$z = xy$$

(25 marks)

3. (a) Maximize $2x+3y+5z$ on the sphere $x^2+y^2+z^2=19$.

(25 marks)

(b) Calculate Δu and du for

$$u = x^2-3xy+2y^2 \text{ at } x=2,\ y=-3,\quad \Delta x=-0.3,\quad \Delta y=0.2.$$

(25 marks)

(c) Find the most general function with the given gradient.

$$\frac{x}{\sqrt{x^2+y^2}}i+\frac{y}{\sqrt{x^2+y^2}}j.$$

(30 marks)

(d) Evaluate the sum $\sum_{i=1}^{3}\sum_{j=1}^{3}2^{i-1}3^{j+1}$.

(20 marks)

4. Evaluate

(a) $\int_0^1\int_{\sqrt{x}}^1 \sin\left(\frac{y^3+1}{2}\right)dydx$.

(25 marks)

(b) $\int_0^{\frac{\pi}{2}}\int_0^{\sin\theta} r\cos\theta\, drd\theta$.

(25 marks)

(c) $\int_0^1\int_1^{2y}\int_0^x (x+2z)\,dzdxdy$.

(25 marks)

(d) $\int_0^{\frac{\pi}{2}}\int_0^1\int_0^{\sqrt{1-x^2}} x\cos z\,dydxdz$.

(25 marks)

5. (a) The volume of a solid T is given in spherical coordinates. Evaluate the repeated integral

$$\int_0^{2\pi}\int_0^{\pi}\int_0^2 \rho^2\sin\phi\,d\rho d\phi d\theta$$

(30 marks)

(b) Take Ω as the parallelogram bounded by

$$x+y=0,\quad x+y=1,\quad x-y=0,\quad x-y=2.$$

Evaluate

$$\iint_{\Omega}\left(x^2-y^2\right)dxdy.$$

(40 marks)

(c) Calculate $\nabla.\boldsymbol{v}$ and $\nabla\times\boldsymbol{v}$.

$$\boldsymbol{v}(x,y)=x\boldsymbol{i}+y\boldsymbol{j}.$$

(30 marks)

1. (a) Bentukan hasil bagi pembezaan

$$\frac{f(x+h,y)-f(x,y)}{h} \text{ dan } \frac{f(x,y+h)-f(x,y)}{h}, \ (h \neq 0)$$

Kemudian, takrifkan bahawa x dan y tetap dan hitung had $h \to 0$.

$$f(x,y) = 2x^2 - y.$$

(25 markah)

(b) Camkan permukaan dan cari surihan:

$$9x^2 + 4y^2 - 36z = 0.$$

(25 markah)

(c) Cari persamaan untuk darjah lengkung dari f yang mengandungi titik P.

$$f(x,y) = 1 - 4x^2 - y^2; \quad P(0,1).$$

(25 markah)

(d) Hitung terbitan separa:

$$f(x,y,z) = z\sin(x-y).$$

(25 markah)

2. (a) Tunjukkan bahawa, jika $u = \frac{xy}{x+y}$ maka $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$.

(25 markah)

(b) Cari vektor kecerunan pada titik P.

$$f(x,y) = 2x^2 - 3xy + 4y^2 \text{ pada } P(2,3).$$

(25 markah)

(c) Cari $\frac{\partial u}{\partial s}$ dan $\frac{\partial u}{\partial t}$.

$u = x^2 - xy$

di mana $x = s\cos t, \quad y = t\sin s$.

(25 markah)

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(d) Cari titik pada permukaan yang mana satah tangen adalah mengufuk.

$z = xy$.

(25 markah)

3. (a) Maksimumkan $2x+3y+5z$ atas sfera $x^2+y^2+z^2=19$.

(25 markah)

(b) Hitung Δu dan du untuk

$u = x^2 - 3xy + 2y^2$ pada $x=2,\ y=-3,\quad \Delta x=-0.3,\quad \Delta y=0.2$.

(25 markah)

(c) Cari fungsi paling am dengan kecerunan berikut.

$$\frac{x}{\sqrt{x^2+y^2}}i + \frac{y}{\sqrt{x^2+y^2}}j.$$

(30 markah)

(d) Nilaikan hasil jumlah $\sum_{i=1}^{3}\sum_{j=1}^{3} 2^{i-1}3^{j+1}$.

(20 markah)

4. Nilaikan

(a) $$\int_0^1 \int_{\sqrt{x}}^1 \sin\left(\frac{y^3+1}{2}\right) dydx.$$

(25 markah)

(b) $$\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r\cos\theta \, drd\theta.$$

(25 markah)

(c) $$\int_0^1 \int_1^{2y} \int_0^x (x+2z)\, dzdxdy.$$

(25 markah)

(d) $\int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{\sqrt{1-x^2}} x\cos z\,dydxdz$.

(25 markah)

5. (a) Isipadu dari pepejal T diberikan dalam koordinat sfera. Nilaikan kamiran berulang

$$\int_0^{2\pi} \int_0^{\pi} \int_0^2 \rho^2 \sin\phi\, d\rho d\phi d\theta .$$

(30 markah)

(b) Ambil Ω sebagai segiempat selari yang dibatasi oleh

$$x+y=0, \quad x+y=1, \quad x-y=0, \quad x-y=2.$$

Nilaikan

$$\iint_{\Omega} \left(x^2-y^2\right) dxdy .$$

(40 markah)

(c) Hitung $\nabla.\boldsymbol{v}$ dan $\nabla\times\boldsymbol{v}$.

$$\boldsymbol{v}(x,y) = x\boldsymbol{i} + y\boldsymbol{j}.$$

(30 markah)

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